AN APPROXIMATE SOLUTION FOR THE BENDING OF A CYLINDRICAL SHELL WITH TWO LONGITUDINAL FLANGES AND LOADED WITH INTERNAL PRESSURE

by

JOHN ATKINS MAYHALL

B. S., University of Alabama, 1951

A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE COLLEGE
OF AGRICULTURE AND APPLIED SCIENCE

1958

TABLE OF CONTENTS

LIST OF SYMBOLS

a	Mean radius of cylindrical shell.
b	Effective bending length of flange in transverse plane (See Fig. 3).
$C_{1m}, C_{2m}, C_{3m}, C_{4m}$	Coefficients or constants of integration which satisfy boundary conditions.
d	The width of the horizontal flange (See Fig. 4).
$D_{FL} = \frac{td^3}{12} E$	Flexural rigidity of the longitudinal flange in bending in the longitudinal plane (Shown in Fig. 4).
$D_{FT} = \frac{t^3 E}{12(1-\mu^2)}$	Flexural rigidity of the longitudinal flange in bending in the transverse plane (See Fig. 3).
$D_S = h^3 E/12(1-\mu^2)$	Flexural rigidity of the cylindrical shell.
E	Modulus of elasticity.
h	Shell thickness.
i	$\sqrt{-1}$
L	Length of cylinder.
$M_x, M_\phi, M_{x\phi}, M_{\phi x}$	Bending and twisting moments per unit length of section of shell as defined in Fig. 6 (positive directions are shown in Fig. 6).
m	Number designating term of series.
$N_x, N_\phi, M_{x\phi}, N_{\phi x}$	Normal and shear forces per unit length of section of shell as defined in Fig. 6 (positive directions are as shown in Fig. 6).
$N_\phi^1 = q\,a$	Membrane hoop force per unit length of shell.
Q_x, Q_ϕ	Transverse shear forces per unit length as shown in Fig. 6 (positive directions are shown).
q	Internal pressure, lb per square inch.
t	The thickness of the longitudinal flange (See Fig. 3).

U_{FL}	Strain energy due to bending of the longitudinal flange in a longitudinal plane.
U_{FT}	Strain energy due to bending of the longitudinal flange in a transverse plane.
u	Component of displacement in the direction of the x axis.
V_s	Strain energy due to bending of the cylindrical shell.
v	Component of displacement in the y direction.
w	Component of displacement in the z direction.
x, y, z	Rectangular coordinates (See Fig. 2).
α	Root of characteristic equation.
β	Imaginary part of root of characteristic equation.
γ	Real part of root of characteristic equation.

INTRODUCTION

This thesis is concerned with the mathematical determination of the forces, moments, and deformations in a cylindrical shell of particular design subjected to internal pressure. The shell is made up of two halves, one of which is shown in Fig. 1[1]. The two halves are bolted together along the longitudinal flanges, and may be joined to other shells by bolting to the transverse end flanges. This type of construction is used extensively in jet engines, the bolted cylinder being used to house the axial-flow compressor.

Simplifications are made to the general equations which describe the bending of a laterally-loaded cylindrical shell (for small deflections), and the resulting simplified equations are used in conjunction with the principle of least work in order to evaluate the forces, moments, and deflections of the shell.

Although the writer believes that the problem attacked in this thesis has not heretofore been solved, the equations (and simplifications of same) governing the deflection of a laterally-loaded cylindrical shell were taken from Timoshenko's Theory of Plates and Shells, pages 446-449. The strain energy due to bending of the shell and longitudinal flanges is summed and then minimized, yielding expressions for moments, forces, and deflections in the shell.

---

[1] All figures in the Appendices.

DESCRIPTION OF PROBLEM

A sketch of a pressure vessel under consideration is shown in Fig. 1. The shell has transverse flanges at either end, each of which is bolted to adjoining shells. The cylindrical shell is made in halves, bolted together with longitudinal flanges. The shell will need to sustain uniform internal pressure.

The pressure loading will induce bending, shear, and normal forces in the shell. The mathematical determination of such forces and moments will be attempted here.

DERIVATION OF EQUATIONS

The system of differential equations which describes the bending of a laterally-loaded cylindrical shell (for small deflections) is as follows (see Fig. 2 and Fig. 6 for explanation of the coordinate system, forces and moments):[1]

$$a\frac{\partial N_x}{\partial x} + \frac{\partial N_{\phi x}}{\partial \phi} = 0$$

$$\frac{\partial N_\phi}{\partial \phi} + a\frac{\partial N_{x\phi}}{\partial x} - Q_\phi = 0$$

$$a\frac{\partial Q_x}{\partial x} + \frac{\partial Q_\phi}{\partial \phi} + N_\phi + q\,a = 0 \qquad (1\ a,b,c,d,e.)$$

$$a\frac{\partial M_{x\phi}}{\partial x} - \frac{\partial M_\phi}{\partial \phi} + a\,Q_\phi = 0$$

$$\frac{\partial M_{\phi x}}{\partial \phi} + a\frac{\partial M_x}{\partial x} - a\,Q_x = 0.$$

---

[1] Timoshenko, S., Theory of Plates and Shells, 1940, page 440.

The general solution of these equations for the problem under consideration would prove too complex for practical application; hence a simplification will be made.

For large cylindrical shells, the membrane theory gives satisfactory results for portions of a shell at considerable distances from the edges, but the theory will not satisfy the conditions at the boundary. It is logical, therefore, to take the membrane theory as a first approximation and use the more elaborate bending theory to satisfy the conditions at the edges.[1] In applying this latter theory, it must be assumed that no external load is distributed over the shell and that only forces and moments such as are necessary to satisfy the boundary conditions are applied along the edges. The bending produced by such forces can be investigated by using equations (1) after placing q equal to zero in these equations.

In the application considered, the ends $x = 0$, and $x = L$ of the shell (Fig. 2) are supported by the transverse flanges in such a manner that the displacements v and w at the ends virtually vanish. Experiments show in such shells that at a small axial distance from the ends, the bending in the axial planes is negligible, and one can assume $M_x = 0$, and $Q_x = 0$ in the equations (1). One can also neglect the twisting moment $M_{x\phi}$. With these assumptions, the system of equations (1) can be considerably simplified, and the resultant forces and components of displacement can be all expressed in terms of the moment M_ϕ.[2]

---

[1] Loc. Cit., page 446.

[2] Loc. Cit., page 447.

From equation (1d)

$$Q_\phi = \frac{1}{a}\,\frac{\partial M_\phi}{\partial x}\ . \qquad (2)$$

Substituting this in equation (1c) leads to (for $q = 0$)

$$N_\phi = -\frac{\partial Q_\phi}{\partial \phi} = -\frac{1}{a}\,\frac{\partial^2 M_\phi}{\partial \phi^2}\ . \qquad (3)$$

Equations (1a) and (1b) then give

$$\frac{\partial N_{x\phi}}{\partial x} = \frac{1}{a}\left(Q_\phi - \frac{\partial N_\phi}{\partial \phi}\right) = \frac{1}{a^2}\left(\frac{\partial M_\phi}{\partial \phi} + \frac{\partial^3 M_\phi}{\partial \phi^3}\right) \qquad (4)$$

and

$$\frac{\partial^2 N_x}{\partial x^2} = -\frac{1}{a}\,\frac{\partial^2 N_{x\phi}}{\partial \phi\,\partial x} = -\frac{1}{a^3}\left(\frac{\partial^2 M_\phi}{\partial \phi^2} + \frac{\partial^4 M_\phi}{\partial \phi^4}\right)\ . \qquad (5)$$

The components of displacement can also be expressed in terms of M_ϕ and its derivatives. Beginning with the known relations[1]

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{1}{Eh}\left(N_x - \mu N_\phi\right)$$

$$\gamma_{x\phi} = \frac{\partial u}{a\partial \phi} + \frac{\partial v}{\partial x} = \frac{2(1+\mu)}{Eh}\,N_{x\phi}$$

$$\epsilon_\phi = \frac{\partial v}{a\partial \phi} - \frac{w}{a} = \frac{1}{Eh}\left(N_\phi - \mu N_x\right), \qquad (6)$$

---

[1] Loc. Cit., pages 354, 355.

these equations lead to

$$\frac{\partial u}{\partial x} = \frac{1}{Eh}\left(N_x - \mu N_\phi\right)$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{Eh}\left[2(1+\mu)\frac{\partial N_{x\phi}}{\partial x} - \frac{1}{a}\left(\frac{\partial N_x}{\partial \phi} - \mu\frac{\partial N_\phi}{\partial \phi}\right)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{Eh}\left[a\left(\mu\frac{\partial^2 N_x}{\partial x^2} - \frac{\partial^2 N_\phi}{\partial x^2}\right) + 2(1+\mu)\frac{\partial^2 N_{x\phi}}{\partial x\partial\phi}\right]$$

$$- \frac{1}{Eha}\left(\frac{\partial^2 N_x}{\partial\phi^2} - \mu\frac{\partial^2 N_\phi}{\partial\phi^2}\right). \tag{7}$$

Using these expressions together with equations (3), (4), and (5) and using an expression for bending moment which is

$$M_\phi = -\frac{D_s}{a^2}\left(\frac{\partial v}{\partial\phi} + \frac{\partial^2 w}{\partial\phi^2}\right), \tag{8}$$

it is finally obtained for the determination of M_ϕ the following differential equation of eighth order:[1]

$$\frac{\partial^8 M_\phi}{\partial\phi^8} + (2+\mu)a^2\frac{\partial^8 M_\phi}{\partial x^2\partial\phi^6} + 2\frac{\partial^6 M_\phi}{\partial\phi^6} + (1+2\mu)a^4\frac{\partial^8 M_\phi}{\partial x^4\partial\phi^4}$$

$$+ 2(2+\mu)^2\frac{a^4}{2}\frac{\partial^6 M_\phi}{\partial x^4\partial\phi^2} + (2+\mu)a^2\frac{\partial^6 M_\phi}{\partial x^2\partial\phi^4} + \frac{\partial^4 M_\phi}{\partial\phi^4}$$

$$+ \mu a^6\frac{\partial^8 M_\phi}{\partial x^6\partial\phi^2} + (2+\mu)a^2\frac{\partial^4 M_\phi}{\partial x^2\partial\phi^2} + 12(1-\mu^2)\frac{a^6}{h^2}\frac{\partial^4 M_\phi}{\partial x^4} = 0. \tag{9}$$

A particular solution of this equation is afforded by the expression

$$M_\phi = A\,e^{\alpha\phi}\sin\frac{m\pi x}{L}. \tag{10}$$

---

[1] Loc. Cit., 448.

Substituting this expression in equation 9 and using the notation

$$\frac{m\,\pi a}{L} = \lambda,$$

the following algebraic equation for α is obtained:

$$\begin{aligned}\alpha^8 &+ \left[2-(2+\mu)\lambda^2\right]\alpha^6 + \left[(1+2\mu)\,\lambda^4 - 2(2+\mu)\,\lambda^2 + 1\right]\,\alpha^4 \\ &+ \left[-\mu\lambda^6 + (1+\mu)^2\,\lambda^4 - (2+\mu)\,\lambda^2\right]\,\alpha^2 \\ &\qquad + 12(1-\mu^2)\,\frac{a^2}{h^2}\,\lambda^4 = 0.\end{aligned} \tag{11}$$

The eight roots of this equation can be put in the form

$$\alpha_{1,2,3,4} = \pm\,(\gamma_1 \pm i\,\beta_1);\quad \alpha_{5,6,7,8} = \pm\,(\gamma_2 \pm i\,\beta_2). \tag{12}$$

Beginning with the edge $\phi = 0$ (Fig. 2) and assuming that the moment M_ϕ rapidly diminishes as ϕ increases,[1] only those four of the roots (12) which satisfy this requirement are used. Then, combining the four corresponding solutions (10), the following is obtained,

$$M_\phi = \left[e^{-\gamma_1\phi}(C_1 \cos\beta_1\phi + C_2 \sin\beta_1\phi) + e^{-\gamma_2\phi}(C_3 \cos\beta_2\phi + C_4 \sin\beta_2\phi)\right]\,\sin\frac{m\pi x}{L}$$

If, instead of a single term (10), the trigonometric series

[1] In large shells, the bending moment usually fades rapidly with distance from an edge, if loading is due to uniform internal pressure.

$$M_\phi = \sum_{m=1,\ldots}^{\infty} \left[e^{-\gamma_{1m}\phi} \left(C_{1m} \cos \beta_{1m}\phi + C_{2m} \sin \beta_{1m} \phi\right) \right.$$

$$\left. + e^{-\gamma_{2m}\phi} \left(C_{3m} \cos \beta_{2m}\phi + C_{4m} \sin \beta_{2m} \phi\right) \right] \sin \frac{m\pi x}{L} \qquad (13)$$

is taken; then any distribution of the bending moment M_ϕ along the edge $\phi = 0$ can be obtained. Having an expression for M_ϕ, the resultant forces Q_ϕ, N_ϕ, and $N_{x\phi}$ are obtained from equations (2), (3), and (4).

APPLICATION OF THE PRINCIPLE OF LEAST WORK

Now in order to determine the actual distribution of moments and forces in the shell, the principle of least work will be used. This principle can be stated as follows: Of the infinite number of load and moment distributions in the shell which could possibly satisfy the conditions of equilibrium, but which do not necessarily satisfy compatibility, the true or compatible distribution has the least strain energy in the system, all other non-compatible stress distributions having greater energy than the true one.[1] In applying this principle, one needs only to calculate the strain energy in the entire system and minimize it.

To do this, only the energy due to bending will be determined, since it can be shown that energy due to shear, tension, and twist is negligibly small. The energy in the entire system will be broken into three parts

[1] Den Hartog, J. P., Advanced Strength of Materials, 1952, page 212.

as follows: (1) energy due to bending of the cylindrical shell, (2) energy due to the transverse bending of the longitudinal flanges in transverse planes perpendicular to the center line of the cylinder (see Fig. 3), and (3) energy due to bending of the longitudinal flanges in a longitudinal plane containing the center line of the cylinder.

ENERGY DUE TO BENDING OF THE CYLINDRICAL SHELL

Half of the energy, V_s, in the shell will be[1]

$$V_s = \int_0^L \int_0^{\pi/2} \frac{M_\phi^2}{2D_s} \, a \, d\phi \, dx$$

$$= \frac{a}{2D_s} \int_0^L \int_0^{\pi/2} \sum_{m=1,2,3\cdots}^{\infty} \Big[e^{-\gamma_{1m}\phi} (C_{1m} \cos\beta_{1m}\phi + C_{2m} \sin\beta_{1m}\phi)$$

$$+ e^{-\gamma_{2m}\phi} (C_{3m} \cos\beta_{2m}\phi + C_{4m} \sin\beta_{2m}\phi) \Big]^2 \sin^2 \frac{m\pi x}{L} \, . \qquad (14)^2$$

Performing the indicated integration, the following expression is obtained:

---

[1] Since the loading and shell is symmetrical about the axis of the shell, only one half need be considered.

[2] The terms involving $\sin \frac{m\pi x}{L} \; \sin \frac{n\pi x}{L}$, where $m \neq n$, have been omitted since they produce terms which are zero when integrated between the limits 0 and L.

$$V_s = \frac{aL}{4D_s} \sum_{m=1}^{\infty} \Big[I_1 C_{1m}^2 + I_{1.5} C_{3m}^2 + I_2 C_{2m}^2 + I_{2.5} C_{4m}^2 + I_3 C_{1m} C_{2m}$$

$$+ I_{3.5} C_{3m} C_{4m} + (C_{1m} C_{3m} + C_{2m} C_{4m}) I_4$$

$$+(C_{1m} C_{3m} - C_{2m} C_{4m}) I_{4.5} + (C_{2m} C_{3m} - C_{1m} C_{4m}) I_5 \qquad (15)^1$$

$$+ (C_{1m} C_{4m} + C_{2m} C_{3m}) I_{5.5} \Big].$$

Now, in order to minimize the energy subsequently, it will be required to obtain the following quantities:

$$\frac{\partial V_s}{\partial C_{1m}}, \ \frac{\partial V_s}{\partial C_{2m}}, \ \frac{\partial V_s}{\partial C_{3m}}, \ \frac{\partial V_s}{\partial C_{4m}} .$$

These quantities can be found from equation (15) by partially differentiating as follows:

$$\frac{\partial V_s}{\partial C_{1m}} = \frac{aL}{4D_s} \Big[2I_1 C_{1m} + I_3 C_{2m} + (I_4 + I_{4.5}) C_{3m} + (I_{5.5} - I_5) C_{4m} \Big], \quad (16)$$

$$\frac{\partial V_s}{\partial C_{2m}} = \frac{aL}{4D_s} \Big[2I_2 C_{2m} + I_3 C_{1m} + (I_4 - I_{4.5}) C_{4m} + (I_5 + I_{5.5}) C_{3m} \Big], \quad (17)$$

$$\frac{\partial V_s}{\partial C_{3m}} = \frac{aL}{4D_s} \Big[2 I_{1.5} C_{3m} + I_{3.5} C_{4m} + (I_4 + I_{4.5}) C_{1m} + (I_5 + I_{5.5}) C_{2m} \Big], \quad (18)$$

$$\frac{\partial V_s}{\partial C_{4m}} = \frac{aL}{4D_s} \Big[2 I_{2.5} C_{4m} + I_{3.5} C_{3m} + (I_4 - I_{4.5}) C_{2m} + (I_{5.5} - I_5) C_{1m} \Big]. \quad (19)$$

---

[1] For identification of $I_1, I_{1.5}, I_2$, etc. see Appendices, Eq. 1-10.

ENERGY DUE TO THE TRANSVERSE BENDING OF THE LONGITUDINAL FLANGE IN TRANSVERSE PLANES PERPENDICULAR TO THE CENTER LINE OF THE CYLINDER

Consider Fig. 3. The transverse bending of the flange is depicted in Fig. 3-a. In calculating the energy due to bending in the transverse plane, the axial load $Q_{\phi=0}$ will be neglected since it can be shown to have a negligible effect on this bending.

From equations (3) and (13),

$$N_{\phi=0} = -\frac{1}{a}\left.\frac{\partial^2 M_\phi}{\partial\phi^2}\right|_{\phi=0} = -\frac{1}{a}\sum_{m=1}^{\infty}\Big[(\gamma_{1m}^2-\beta_{1m}^2)C_{1m} - 2\gamma_{1m}\beta_{1m}C_{2m}+(\gamma_{2m}^2-\beta_{2m}^2)C_{3m}-2\gamma_{2m}\beta_{2m}C_{4m}\Big]\sin\frac{m\pi x}{L}. \tag{20}$$

The variable bending moment, M, is determined from Fig. 3-b as

$$M_\xi = M_{\phi=0} + \xi N_{\phi=0} + \xi a q + \frac{\xi^2 q}{2}.$$

From equation 13,

$$M_{\phi=0} = \sum_{m=1}^{\infty}(C_{1m}+C_{3m})\sin\frac{m\pi x}{L}. \tag{21}$$

Let the bending energy in the flange in a transverse plane be U_{FT}, where

$$U_{FT} = \frac{1}{2D_{FT}}\int_0^L\int_0^b M_\xi^2\, d_\xi \quad dx$$

$$= \frac{1}{2D_{FT}}\int_0^L\Bigg[\int_0^b\Bigg\{\sum_{m=1}^{\infty}(C_{1m}+C_{3m})\sin\frac{m\pi x}{L} - \frac{\xi}{a}\sum_{m=1}^{\infty}\Big[(\gamma_{1m}^2-\beta_{1m}^2)C_{1m}$$

$$- 2\gamma_{1m}\beta_{1m} C_{2m} + (\gamma^2_{1m} - \beta^2_{1m}) C_{3m} - 2\gamma_{2m}\beta_{2m} C_{4m} \Bigg] \sin \frac{m\pi x}{L}$$

$$+ q a \xi + q \xi^2/2 \Bigg\}^2 d\xi \Bigg] dx \cdot \qquad (22)$$

Performing the indicated integration and partially differentiating, the following is obtained:

$$\frac{\partial U_{FT}}{\partial C_{1m}} = \frac{1}{D_{FT}} \Bigg\{ \frac{bL}{2} \left[1 - \frac{b}{2a} (\gamma^2_{1m} - \beta^2_{1m}) \right] (C_{1m} + C_{3m})$$

$$- \frac{b^2 L}{2a} \left[\frac{1}{2} - \frac{b}{3a} (\gamma^2_{1m} - \beta^2_{1m}) \right] \Bigg[(\gamma^2_{1m} - \beta^2_{1m}) C_{1m} \qquad (23)$$

$$- 2\gamma_{1m}\beta_{1m} C_{2m} + (\gamma^2_{2m} - \beta^2_{2m}) C_{3m} - 2\gamma_{2m}\beta_{2m} C_{4m} \Bigg] \Bigg\}$$

$$+ \frac{q\, 2L}{D_{FT}\pi m} \left[\frac{1 + (-1)^{m+1}}{2} \right] \Bigg\{ a b^2 \left[\frac{1}{2} - \frac{b}{3a} (\gamma^2_{1m} - \beta^2_{1m}) \right]$$

$$+ \frac{b^3}{2} \left[\frac{1}{3} - \frac{b}{4a} (\gamma^2_{1m} - \beta^2_{1m}) \right] \Bigg\} .$$

$$\frac{\partial U_{FT}}{\partial C_{2m}} = \frac{2\gamma_{1m}\beta_{1m}b^2}{a\,D_{FT}} \left\{ \frac{L}{4}(C_{1m}+C_{3m}) - \frac{bL}{6a}\left[(\gamma^2_{1m} - \beta^2_{1m})\,C_{1m} \right.\right.$$

$$\left. - 2\gamma_{1m}\beta_{1m}C_{2m} + (\gamma^2_{2m} - \beta^2_{2m})\,C_{3m} - 2\gamma_{2m}\beta_{2m}C_{4m} \right] \tag{24}$$

$$\left. + \frac{q\,L\,b}{\pi m}\left[1 + (-1)^{m+1} \right] \left(\frac{a}{3} + \frac{b}{8} \right) \right\},$$

$$\frac{\partial U_{FT}}{\partial C_{3m}} = \frac{bL}{2D_{FT}} \left\{ \left[1 - \frac{b}{2a}(\gamma^2_{2m} - \beta^2_{2m}) \right] (C_{1m} + C_{3m}) \right.$$

$$- \frac{b}{a}\left[\frac{1}{2} - \frac{b}{3a}(\gamma^2_{2m} - \beta^2_{2m}) \right] \left[(\gamma^2_{1m} - \beta^2_{1m})\,C_{1m} \right. \tag{25}$$

$$\left.\left. - 2\gamma_{1m}\beta_{1m}C_{2m} + (\gamma^2_{2m} - \beta^2_{2m})C_{3m} - 2\gamma_{2m}\beta_{2m}C_{4m} \right] \right\}$$

$$+ \frac{q\,b^2\,L}{D_{FT}\,\pi m}\left[1 + (-1)^{m+1} \right] \left\{ a\left[\frac{1}{2} - \frac{b}{3a}(\gamma^2_{2m} - \beta^2_{2m}) \right] \right.$$

$$\left. + \frac{b}{2}\left[\frac{1}{3} - \frac{b}{4a}(\gamma^2_{2m} - \beta^2_{2m}) \right] \right\},$$

$$\frac{\partial U_{FT}}{\partial C_{4m}} = \frac{2\gamma_{2m}\beta_{2m} b^2 L}{a D_{FT}} \left\{ \frac{1}{4} (C_{1m} + C_{3m}) - \frac{b}{6a} \left[(\gamma^2_{1m} - \beta^2_{1m}) C_{1m} \right.\right.$$

$$\left. - 2\gamma_{1m}\beta_{1m} C_{2m} + (\gamma^2_{2m} - \beta^2_{2m}) C_{3m} - 2\gamma_{2m}\beta_{2m} C_{4m} \right] \tag{26}$$

$$\left. + \frac{q\, b}{\pi m} \left(\frac{a}{3} + \frac{b}{8} \right) \left[1 + (-1)^{m+1} \right] \right\} .$$

Thus equations (21) - (24) define the rate of change of the strain energy (due to transverse bending of the flanges) with respect to the four coefficients C_{1m}, C_{2m}, C_{3m}, and C_{4m}.

ENERGY DUE TO BENDING OF THE LONGITUDINAL FLANGE IN A PLANE CONTAINING THE CENTER LINE OF THE CYLINDER

The strain energy due to bending of the longitudinal flange in the longitudinal plane (Fig. 4) will be denoted by U_{FL}, where

$$U_{FL} = \frac{D_{FL}}{2} \int_0^L \left(\frac{d^2 w}{dx^2} \right)^2_{FL} dx ,$$

where $\left.\frac{d^2 w}{dx^2}\right|_{FL}$ is developed in Appendices (see equation v); from this

$$U_{FL} = \frac{2D_{FL}}{2(Eh)^2 a^8} \int_0^{L/2} \left[\sum_{m=1}^{\infty} (K_{1m} C_{1m} + K_{2m} C_{2m} \right.$$

$$\left. + K_{3m} C_{3m} + K_{4m} C_{4m}) \sin \frac{m\pi x}{L} \right]^2 dx. \tag{27}$$

Now, the rate of change of U_{FL} with respect to C_{1m}, C_{2m}, C_{3m}, and C_{4m} will be found.

$$\frac{\partial U_{FL}}{\partial C_{1m}} = \frac{D_{FL}}{(Eh)^2 a^8} \int_0^L K_{1m} (K_{1m} C_{1m} + K_{2m} C_{2m} + K_{3m} C_{3m} + K_{4m} C_{4m}) \sin^2 \frac{m \pi x}{L} \, dx \tag{28}$$

$$= \frac{D_{FL} L}{2(Eh)^2 a^8} K_{1m} (K_{1m} C_{1m} + K_{2m} C_{2m} + K_{3m} C_{3m} + K_{4m} C_{4m}).$$

$$\frac{\partial U_{FL}}{\partial C_{2m}} = \frac{D_{FL} L}{2(Eh)^2 a^8} K_{2m} (K_{1m} C_{1m} + K_{2m} C_{2m} + K_{3m} C_{3m} + K_{4m} C_{4m}). \tag{29}$$

$$\frac{\partial U_{FL}}{\partial C_{3m}} = \frac{K_{3m}}{K_{2m}} \frac{\partial U_{FL}}{\partial C_{2m}} . \tag{30}$$

$$\frac{\partial U_{FL}}{\partial C_{4m}} = \frac{K_{4m}}{K_{2m}} \frac{\partial U_{FL}}{\partial C_{2m}} . \tag{31}$$

SUMMARY OF TOTAL STRAIN ENERGY

Now the total strain energy will be summed and minimized. From equations (15), (22), and (27), the total strain energy U_T is

$$U_T = V_s + U_{FT} + U_{FL}. \tag{32}$$

In order that the strain energy in the system be minimum, the following equations must be valid:

$$\frac{\partial U_T}{\partial C_{1m}} = 0$$

$$\frac{\partial U_T}{\partial C_{2m}} = 0$$

$$\frac{\partial U_T}{\partial C_{3m}} = 0 \tag{33}$$

$$\frac{\partial U_T}{\partial C_{4m}} = 0 \; .$$

From equations (16) through (19), (23) through (26), and (28) through (31), $\frac{\partial U_T}{\partial C_{1m}}$, $\frac{\partial U_T}{\partial C_{2m}}$, $\frac{\partial U_T}{\partial C_{3m}}$, and $\frac{\partial U_T}{\partial C_{4m}}$ can be determined.

$$\frac{\partial U_T}{\partial C_{1m}} = \frac{a L}{4D_s} \Big[2 I_1 C_{1m} + I_3 C_{2m} + (I_4 + I_{4.5}) C_{3m}$$

$$+ (I_{5.5} - I_5) C_{4m} \Big] + \frac{b L}{2D_{FT}} \Big\{ \Big[1 - \frac{b}{2a} (\gamma^2_{1m} - \beta^2_{1m}) \Big] (C_{1m}$$

$$+ C_{3m}) - \frac{b}{a} \Big[\frac{1}{2} - \frac{b}{3a} (\gamma^2_{1m} - \beta^2_{1m}) \Big] \Big[(\gamma^2_{1m} - \beta^2_{1m}) C_{1m}$$

$$- 2\gamma_{1m}\beta_{1m} C_{2m} + (\gamma^2_{2m} - \beta^2_{2m}) C_{3m} - 2\gamma_{2m} \beta_{2m} C_{4m} \Big] \Big\}$$

$$+ \frac{q L b^2}{\pi m D_{FT}} \Big[1 + (-1)^{m+1} \Big] \Big\{ a \Big[\frac{1}{2} - \frac{b}{3a} (\gamma^2_{1m} - \beta^2_{1m}) \Big]$$

$$+ \frac{b}{2} \Big[\frac{1}{3} - \frac{b}{4a} (\gamma^2_{1m} - \beta^2_{1m}) \Big] \Big\} + \frac{D_{FL} L}{2(Eh)^2 a^8} K_{1m} (K_{1m} C_{1m}$$

$$+ K_{2m}C_{2m} + K_{3m}C_{3m} + K_{4m}C_{4m}).$$

Letting

$$
\begin{aligned}
&1 - \frac{b}{2a}(\gamma^2_{1m} - \beta^2_{1m}) = F_1\,, && N_1 = \frac{1}{3} - \frac{b}{4a}(\gamma^2_{1m} - \beta^2_{1m}),\\
&1 - \frac{b}{2a}(\gamma^2_{2m} - \beta^2_{2m}) = F_2\,, && N_2 = \frac{1}{3} - \frac{b}{4a}(\gamma^2_{2m} - \beta^2_{2m}), \qquad (34)\\
&\frac{1}{2} - \frac{b}{3a}(\gamma^2_{1m} - \beta^2_{1m}) = G_1\,, && \gamma^2_{1m} - \beta^2_{1m} = H_1\,,\quad J_1 = 2\gamma_{1m}\beta_{1m},\\
&\frac{1}{2} - \frac{b}{3a}(\gamma^2_{2m} - \beta^2_{2m}) = G_2\,, && \gamma^2_{2m} - \beta^2_{2m} = H_2\,,\quad J_2 = 2\gamma_{2m}\beta_{2m},
\end{aligned}
$$

the first of equations (33) becomes

$$
\begin{aligned}
&\left[\frac{aI_1}{2D_s} + \frac{b}{2D_{FT}}\left(F_1 - \frac{b}{a}G_1H_1\right) + \frac{D_{FL}K^2_{1m}}{2(Eh)^2a^8}\right]C_{1m} + \left(\frac{aI_3}{4D_s}\right.\\
&\left.+\frac{b^2J_1G_1}{2a\,D_{FT}} + \frac{D_{FL}K_{1m}K_{2m}}{2(Eh)^2a^8}\right)C_{2m} + \left[\frac{a}{4D_s}(I_4 + I_{4.5})\right. \qquad (33a)\\
&\left.+\frac{b}{2D_{FT}}\left(F_1 - \frac{b}{a}G_1H_2\right) + \frac{D_{FL}K_{1m}K_{3m}}{2(Eh)^2a^8}\right]C_{3m} + \left[\frac{a}{4D_s}(I_{5.5} - I_5)\right.\\
&\left.+\frac{b^2J_2G_1}{2aD_{FT}} + \frac{D_{FL}K_{1m}K_{4m}}{2(Eh)^2a^8}\right]C_{4m} = -\frac{qb^2}{\pi D_{FT}}\left[\frac{1+(-1)^{m+1}}{m}\right]\left(aG_1 + \frac{b}{2}N_1\right).
\end{aligned}
$$

Also $\dfrac{\partial U_T}{\partial C_{2m}} = \dfrac{\partial V_s}{\partial C_{2m}} + \dfrac{\partial U_{FT}}{\partial C_{2m}} + \dfrac{\partial U_{FL}}{\partial C_{2m}} = 0\,,$

or simplifying, one is led to

$$\left[\frac{aI_3}{4D_s} + \frac{J_1 b^2}{2aD_{FT}}\left(\frac{1}{2} - \frac{bH_1}{3a}\right) + \frac{D_{FL} K_{1m} K_{2m}}{2(Eh)^2 a^8}\right] C_{1m} + \left[\frac{aI_2}{2D_s} + \frac{J_1^2 b^3}{6a^2 D_{FT}}\right.$$

$$\left. + \frac{D_{FL} K^2_{2m}}{2(Eh)^2 a^8}\right] C_{2m} + \left[\frac{a}{4D_s}(I_5 + I_{5.5}) + \frac{J_1^2 b^2}{2aD_{FT}}\left(\frac{1}{2} - \frac{bH_2}{3a}\right)\right.$$

$$\left. + \frac{D_{FL} K_{2m} K_{3m}}{2(Eh)^2 a^8}\right] C_{3m} + \left[\frac{a}{4D_s}(I_4 - I_{4.5}) + \frac{J_1 J_2 b^3}{6a^2 D_{FT}}\right.$$

$$\left. + \frac{D_{FL} K_{2m} K_{4m}}{2(Eh)^2 a^8}\right] C_{4m} = -\frac{q\, b^3 J_1}{\pi a D_{FT}} \quad \frac{a}{3} + \frac{b}{8} \quad \left[\frac{1 + (-1)^{m+1}}{m}\right]. \tag{35}$$

Also,

$$\frac{\partial U_T}{\partial C_{3m}} = \frac{\partial V_s}{\partial C_{3m}} + \frac{\partial U_{FT}}{\partial C_{3m}} + \frac{\partial U_{FL}}{\partial C_{3m}} = 0,$$

or simplifying, this becomes

$$\left[\frac{a}{4D_s}(I_4 + I_{4.5}) + \frac{b}{2D_{FT}}\left(F_2 - \frac{b H_1 G_2}{a}\right) + \frac{D_{FL} K_{1m} K_{3m}}{2(Eh)^2 a^8}\right] C_{1m}$$

$$+ \left[\frac{a}{4D_s}(I_5 + I_{5.5}) + \frac{b^2 J_1 G_2}{2 a D_{FT}} + \frac{D_{FL} K_{2m} K_{3m}}{2(Eh)^2 a^8}\right] C_{2m} + \left[\frac{a}{2D_s} I_{1.5}\right.$$

$$\left. + \frac{b}{2D_{FT}}\left(F_2 - \frac{b}{a} G_2 H_2\right) + \frac{D_{FL} K^2_{3m}}{2(Eh)^2 a^8}\right] C_{3m} + \left[\frac{a}{4D_s} I_{3.5}\right.$$

$$\left. + \frac{D_{FL} K_{3m} K_{4m}}{2(Eh)^2 a^8}\right] C_{4m} = -\frac{qb^2}{\pi D_{FT}}\left(a G_2 + \frac{b}{2} N_2\right)\left[\frac{1 + (-1)^{m+1}}{m}\right]. \tag{36}$$

And finally,

$$\frac{\partial U_T}{\partial C_{4m}} = \frac{\partial V_s}{\partial C_{4m}} + \frac{\partial U_{FT}}{\partial C_{4m}} + \frac{\partial U_{FL}}{\partial C_{4m}} = 0,$$

or from equations (19), (26), (31), and (34), the following is obtained,

$$\left[\frac{a}{4D_s}(I_{5.5} - I_5) + \frac{J_2 b^2}{2aD_{FT}}\left(\frac{1}{2} - \frac{b}{3a} H_1\right) + \frac{D_{FL} K_{1m} K_{4m}}{2(Eh)^2 a^8}\right] C_{1m}$$

$$+ \left[\frac{a}{4D_s}(I_4 - I_{4.5}) + \frac{J_2 b^3 J_1}{6a^2 D_{FT}} + \frac{D_{FL} K_{2m} K_{4m}}{2(Eh)^2 a^8}\right] C_{2m} + \left[\frac{aI_{3.5}}{4D_s}\right.$$

$$\left. + \frac{J_2 b^2}{2aD_{FT}}\left(\frac{1}{2} - \frac{b}{3a} H_2\right) + \frac{D_{FL} K_{3m} K_{4m}}{2(Eh)^2 a^8}\right] C_{3m} + \left(\frac{aI_{2.5}}{2D_s} + \frac{J_2^2 b^3}{6a^2 D_{FT}}\right.$$

$$\left. + \frac{D_{FL} K_{4m}^2}{2(Eh)^2 a^8}\right) C_{4m} = -\frac{qJ_2 b^3}{a\pi D_{FT}}\left(\frac{a}{3} + \frac{b}{8}\right)\left[\frac{1 + (-1)^{m+1}}{m}\right]. \qquad (37)$$

DETERMINATION OF MOMENTS AND FORCES IN THE SHELL AND FLANGES

Now in order to determine the bending moment M_ϕ, the normal forces N_ϕ and N_x, the shear forces $N_{\phi x}$ and $N_{x\phi}$, and the transverse shear force Q_ϕ, one needs first to solve for the negative roots of equation (11) for various values of $m(m = 1, 2, 3, \ldots)$ or for the four roots

$$\alpha_{1,3} = -(\gamma_1 \pm i\beta), \; \alpha_{5,7} = -(\gamma_2 \pm i\beta_2).$$

Then with these values, one can solve for the coefficients of C_{1m}, C_{2m},

C_{3m}, and C_{4m} in equations (33a), (35), (36), and 37. Having the coefficients, one can solve for C_{1m}, C_{2m}, C_{3m}, and C_{4m} by solving equations (33a), (35), (36), and (37) simultaneously. It is obvious from these four equations that the equations become homogeneous for even values of m, and, consequently, $C_{1m} = C_{2m} = C_{3m} = C_{4m} = 0$ for even values of m.

After obtaining the values of the coefficients C_{1m}, C_{2m}, C_{3m}, and C_{4m}, it is an easy matter to determine M_ϕ from equation (13), Q_ϕ from equation (2), N_ϕ from equation (3), and $N_{x\phi}$ from equation (4). It must be remembered, however, that forces, N'_ϕ, determined from the membrane solution must be superposed on those determined from the above edge-bending solution, where

$$N'_\phi = q\,a\;.$$

DETERMINATION OF RADIAL DEFLECTION OF THE SHELL

The radial deflection obtained from the membrane solution must be superposed on that obtained from the bending solution. The deflection obtained from the membrane theory is w_m, where

$$w_m = -\,q\,\frac{a^2}{Eh}\;. \tag{38}$$

The radial deflection obtained from the bending theory can be obtained from integration of equation (iv) in the Appendices as follows:

$$w_B = -\frac{L^3}{\pi E h a^4} \sum_{m=1,3,5,\ldots}^{\infty} (K_{1m} C_{1m} + K_{2m} C_{2m} + K_{3m} C_{3m}$$

$$+ K_{4m} C_{4m}) \frac{1}{m^2} \sin \frac{m\pi x}{L} . \qquad (39)^1$$

The total radial deflection will be w_t, where

$$w_t = w_m + w_B . \qquad (40)$$

NUMERICAL EXAMPLE

A numerical solution for the moment and force distribution in a cylindrical shell was attempted. The dimensions of the shell were as follows (Figs. 2 and 3):[2]

$L = 32.0$, $a = 9.70$, $h = .204$, $b = .705$, $t = .662$, $\mu = .3$, $c = 1.08$,

$q = 100$ psi,

$D_s = 1.789(10)^{-4}E$, $D_{ft} = .02657\ E$, $D_{fl} = .06949\ E$, $E = 10(10)^6$ psi.

Equation (11) was solved for the negative roots for values of m from 1 to 9; these values were found to be as follows:

m	γ_1	β_1	γ_2	β_2
1	3.195	1.314	1.318	3.187
3	5.860	2.141	2.418	5.194
5	7.906	2.622	3.260	6.366
7	9.745	2.941	4.012	7.144
9	11.489	3.155	4.720	7.664

[1] This expression yields the radial deflection at the edge $\phi = 0$ only.

[2] All dimensions are in inches.

Equation (11) was solved by the use of Underwood's Elecom 120 digital computer, and the accuracy obtained was very good.[1]

I_1, $I_{1.5}$, $I_{2.5}$, I_3, $I_{3.5}$, I_4, $I_{4.5}$, and $I_{5.5}$ (see Appendices) were computed by the use of a Burroughs E101 computer.[2] These values we re found to be as shown in Table 1.

The K values (see Appendices) were also evaluated by the use of the E101 computer. These values are shown in Table 2.

The coefficients of C_{1m}, C_{2m}, C_{3m}, and C_{4m} in equations (33a), (34), (35), and (36) were also calculated with the E101 computer. These were found to be as shown in Table 3. Equations (33a), (34), (35), and (36) were then solved simultaneously for C_{1m}, C_{2m}, C_{3m}, and C_{4m} by use of the Elecom 120. These values were found to be as follows:

m	C_{1m}	C_{2m}	C_{3m}	C_{4m}	
1	68.482	-175.332	-49.886	24.438	
3	36.814	- 78.788	-24.596	8.368	
5	16.724	- 49.836	-11.691	12.493	5.20
7	7.519	- 18.817	- 4.859	2.265	
9	4.297	- 11.998	- 2.742	1.536	
11	2.36	- 6.40	- 1.55	.66	

C_{1m}, C_{2m}, C_{3m}, and C_{4m} versus m were plotted on semi-log paper, and it was found that C_{1m}, C_{2m}, and C_{3m} plotted as straight lines. At the point where $m = 5$, however, there was a large discontinuity in the curve of C_{4m} versus m. The calculations (involving C_{4m}) were exhaustively checked, but no error was found. In order for the plot of C_{4m} versus m to form a smooth curve, it was found that C_{4m} should

[1] The Elecom 120 computer has a storage capacity of 1000 words.

[2] The E101 computer has a storage capacity of 100 words.

be 5.20 at m = 5. It is disturbing that only one curve has a point of discontinuity; this seemed to indicate that the C values for a large number of m values should be calculated in order to insure convergence of the solution. Values of C_{1m}, C_{2m}, C_{3m}, and C_{4m} were determined by extrapolating the C - versus - m plot on semi-log paper, in spite of the possible danger of discontinuity. These values are shown above. This was done in order to improve convergence.

$M_{\phi=0}$, $N_{\phi=0}$, and $Q_{\phi=0}$ were calculated from equations (21), (20), and (2) respectively. These values were plotted versus $\frac{x}{L}$ and are shown in Fig. 7.

In order to check the solution, the slope, $\left.\frac{\partial w}{a\,\partial\phi}\right|_{\phi=0}$, of the shell at the junction of the shell and flange was calculated. To accomplish this, equation (i) in the Appendices was integrated four times with respect to x and then differentiated with respect to ϕ. This lengthy manipulation results in

$$\left.\frac{\partial w}{a\partial\phi}\right|_{\phi=0} = \frac{L^2}{Eh\,a^3\,\pi^2}\sum_{m=1,3,5,\cdots}^{\infty}\frac{1}{m^2}\Big[(\gamma_{1m}K_{1m}+\beta_{1m}K_{2m})\,C_{1m} + (\gamma_{1m}K_{2m}-\beta_{1m}K_{1m})\,C_{2m} + (\gamma_{2m}K_{3m} + \beta_{2m}K_{4m})\,C_{3m} + (\gamma_{2m}K_{4m}-\beta_{2m}K_{3m})\,C_{4m}\Big]\sin\frac{m\pi x}{L}. \quad (41)$$

$\left.\frac{\partial w}{a\,\partial\phi}\right|_{\phi=0}$ was calculated and plotted versus $\frac{x}{L}$ and is shown in Fig. 7.

DISCUSSION

The plot shown in Fig. 7 of $M_{\phi=0}$, $Q_{\phi=0}$, and $N_{\phi=0}$ indicates fairly good convergence for these values. Apparently several more terms of the series should have been used in order to get smooth curves (the dotted lines in Fig. 7 show actual values calculated, whereas the solid lines show average values). The reason that more terms of the series (series in equations (21), (20), (2), and (41)) were not used was because of scaling problems with the above-mentioned computers. In all, 22 separate programs were written and "debugged" for the E101 computer, and each program had scaling problems. The scaling range was exceeded after $m = 9$, and the writer had not the time to rewrite the programs. The whole analysis could be programmed in floating point on the IBM 704 which would eliminate scaling problems, but this would not be justified until all errors are definitely eliminated.

The signs of all values calculated seemed right except that of $M_{\phi=0}$ and $\left.\frac{\partial w}{a\partial\phi}\right|_{\phi=0}$. In calculating these two quantities, however, large numbers which were almost equal were subtracted. This indicated that the correct degree of accuracy was not obtained with the E101 computer; i.e., in the automatic computing sequence, possibly significant figures were shifted out of memory and lost. This again is a scaling problem, and it is apparent that a hand calculation is out of the question, since it would be humanly impossible to make the number of calculations required without error.

Exhaustive checking was done on all calculations made, and only one minor error was discovered and corrected. One thing which indicated that fairly accurate results were obtained was that, although the coefficients listed in Table 3 were calculated independently, the following reciprocal principle held:

$$\alpha_{ij} = \alpha_{ji} \, .$$

This can be noted in Table 3.

To further check the accuracy obtained, the change in slope at the junction of the shell and flange was determined by the analysis indicated in Fig. 3; i.e., by calculating the slope change due to the loads exerted on the flange. This was done for $\frac{x}{L} = .5$ using the following loads (taken from Fig. 7):

$$M_{\phi=0} = 9.70 \frac{\text{in.-lb}}{\text{in.}} , \quad Q_{\phi=0} = -14.2 \frac{\text{lb}}{\text{in.}} , \quad N_{\phi=0} = -50 \frac{\text{lb}}{\text{in.}} \quad .$$

The slope will be $\left. \frac{dy}{d\zeta} \right|_{\zeta=0}$, where

$$\left. \frac{dy}{d\zeta} \right|_{\zeta=0} = \frac{b}{D_{FT}} \left[M_{\phi=0} + \frac{qb}{2} \left(a + \frac{b}{3}\right) + N_{\phi=0} \frac{b}{2} \right] ,$$

$= .000908$ radians in this case.

This change in slope is opposite in direction but equal in magnitude to that indicated in Fig. 7.

If one were to assume the cylindrical shell infinite in length and a uniform moment distributed along the side $\phi = 0$, one would find that a moment of M in.-lb would be needed to correct this incompatible change

in slope, where

$$M = -(.000908 + .000907)\frac{2D_s}{\pi a}$$

or $M = -.213 \frac{\text{in.-lb}}{\text{in.}}$ in this case. Hence, although the incompatible change in slope appears to be a serious error, it apparently is only the result of inadvertent loss of significant digits in the use of a small digital computer. Hence, additional work needs to be done in programming more carefully, possibly using a larger computer. Unfortunately, the writer had neither the time nor the facilities required to accomplish this.

CONCLUSIONS

An approximate solution for a cylindrical shell with two longitudinal flanges loaded by uniform internal pressure has been presented by perhaps using the only possible method - the theorem of least work. Although this energy method is probably the most powerful tool available for the analysis of such problems as presented in this paper, it appears also to be probably the most laborious tool one could use - as is evident from the almost astronomical number of calculations which were necessary in the solution presented here. Only the use of automatic computers can afford such a solution as has been made in this thesis, and the whole analysis could be performed by a single program using a computer comparable to the IBM 704.

The analysis presented is valid only for points in the shell at distances of three inches or more from the ends because of the end effects on the bending of the shell. The reasons for this are explained in detail in the

Appendices. These end effects render the analysis invalid because the shell was assumed simply supported and because the resistance of the horizontal flanges to the shell deflection (due to the uniform internal pressure) was neglected. As shown in the Appendices, these effects dampen out at distances of approximately three inches from the ends.

The analysis was performed essentially by evaluating the first five terms of the various series defining $M_{\phi=0}$, $N_{\phi=0}$, $Q_{\phi=0}$, etc. (e.g. see equations (13), (20), and (41). Figure 7 indicates the degree of convergence of the solution, the dotted lines showing the actual magnitudes calculated. It would appear that three more terms would provide good convergence of the calculations; i.e., the calculations should at least be carried through to $m = 15$ in the various series.

Poor accuracy was apparently attained in the calculation of $M_{\phi=0}$. Exhaustive checking of both the theoretical and numerical work failed to turn up anything but small errors which changed the result only slightly. Since the calculation of $M_{\phi=0}$ involved subtracting large numbers of the same relative magnitude, it is believed that not enough attention was paid to scaling problems involved in the use of the automatic computers, significant digits being inadvertently lost as a result. More work with a larger computer would have rectified this problem, but unfortunately the writer had neither the time nor the facilities available.

An incompatible change in slope at the junction of the flange and shell evolved in the numerical solution. Since the theorem of least work assures that compatibility be satisfied at all points within the system, this is an apparent paradox - since the junction of the shell and flange represents points internal in the system. Exhaustive checking of both the develop-

ment of equations and the numerical calculations failed to reveal anything other than minor discrepancies (which were corrected). A detailed search revealed that the discrepancy most probably was the result of scaling problems involved in the use of the digital computer. In fixed-decimal-point calculations, significant digits were apparently shifted out and lost, and since equation (41) (the equation for change in slope) involved adding and subtracting almost equal numbers, inaccuracy in the final result occurred. This loss of significant digits could easily occur in any or all of the twenty-two separate programs written for the E101 computer. An exhaustive study of significant digits in the mountainous calculations would have to be done, and appropriate scaling (that would insure that significance would not be lost) of all computer programs would be required. An easier approach would be to use a larger computer with floating-decimal-point calculation. Unfortunately, the writer had neither the time nor facilities available for such an approach.

Although this paper has not presented a usable solution to the designer, it is believed that only more numerical work would be required to verify the numerical technique required for a complete solution. To be in usable form, the solution would need to be programmed on a large computer with input data easily inserted.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. M. E. Raville and Dr. P. G. Kirmser for their patient and valuable instruction and advice concerning the development and solution of the equations involved in this thesis.

LITERATURE CITED

Den Hartog, J. P.,
Advanced strength of materials. New York: McGraw-Hill, 1952.

Timoshenko, S.
Theory of plates and shells, New York: McGraw-Hill, 1940.

APPENDICES

Development of $\left.\frac{d^2w}{dx^2}\right._{FL}$

Differentiate the last of equations (7) with respect to x, as follows:

$$\frac{\partial^2 \frac{\partial^2 w}{\partial x^2}}{\partial x^2} = \frac{\partial^4 w}{\partial x^4} = \frac{1}{Eh}\left[a\left(\mu\frac{\partial^4 N_x}{\partial x^4} - \frac{\partial^4 N_\phi}{\partial x^4}\right) + 2(1+\mu)\frac{\partial^4 N_{x\phi}}{\partial x^3\partial\phi}\right.$$

$$\left. - \frac{1}{a}\left(\frac{\partial^4 N_x}{\partial\phi^2\partial x^2} - \mu\frac{\partial^4 N_\phi}{\partial x^2\partial\phi^2}\right)\right].$$

Now, substituting the expressions for N_x, N_ϕ, $N_{x\phi}$ from equations (5), (3), and (4) in terms of M_ϕ in the above equation leads to

$$\frac{\partial^4 w}{\partial x^4} = \frac{1}{Eha^4}\left[\frac{\partial^6 M_\phi}{\partial\phi^6} + 2a^2\frac{\partial^6 M_\phi}{\partial x^2\partial\phi^4} + a^4\frac{\partial^6 M_\phi}{\partial x^4\partial\phi^2} + \frac{\partial^4 M_\phi}{\partial\phi^4} + (2+\mu)a^2\frac{\partial^4 M_\phi}{\partial x^2\partial\phi^2}\right]. \quad \text{(i)}$$

Now, substituting equation (13) in (i), (for ϕ , 0) the following is obtained:

$$\left.\frac{\partial^4 w}{\partial x^4}\right|_{\phi=0} = \frac{1}{Eh\,a^4}\sum_{m=1}^{\infty}\left[\left[\gamma^6_{1m} + 15\gamma^2_{1m}\,\beta^2_{1m}\left(\beta^2_{1m} - \gamma^2_{1m}\right) - \beta^6_{1m}\right]C_{1m}\right.$$

$$+ 2\gamma_{1m}\beta_{1m}\left[10\gamma^2_{1m}\beta^2_{1m} - 3\left(\gamma^4_{1m} + \beta^4_{1m}\right)\right]C_{2m}$$

$$+ \left[\gamma^6_{2m} + 15\gamma^2_{2m}\,\beta^2_{2m}\left(\beta^2_{2m} - \gamma^2_{2m}\right) - \beta^6_{2m}\right]C_{3m}$$

$$\left. + 2\gamma_{2m}\,\beta_{2m}\left[10\gamma^2_{2m}\,\beta^2_{2m} - 3\left(\gamma^4_{2m} + \beta^4_{2m}\right)\right]C_{4m}\right]\sin\frac{m\pi x}{L}$$

$$- \frac{2a^2\pi^2}{L^2} m^2 \Big[(\gamma^4_{1m} - 6\gamma^2_{1m}\beta^2_{1m} + \beta^4_{1m}) C_{1m}$$

$$- 4\gamma_{1m}\beta_{1m} (\gamma^2_{1m} - \beta^2_{1m}) C_{2m} + (\gamma^4_{2m} - 6\gamma^2_{2m}\beta^2_{2m} + \beta^4_{2m}) C_{3m}$$

$$- 4\gamma_{2m}\beta_{2m}(\gamma^2_{2m}\beta_{2m})C_{4m} \Big] \left(1 - \frac{L^2}{2a^2\pi^2 m^2}\right) \sin \frac{m\pi x}{L}$$

$$+ \Big[(\gamma^2_{1m} - \beta^2_{1m}) C_{1m} - 2\gamma_{1m}\beta_{1m} C_{2m}$$

$$+ (\gamma^2_{2m} - \beta^2_{2m}) C_{3m} - 2\gamma_{2m}\beta_{2m} C_{4m} \Big]$$

$$\left[\frac{a^4\pi^4 m^4}{L^4} - \frac{2(1+\frac{\mu}{2})\, a^2\pi^2 m^2}{L^2} \right] \sin \frac{m\pi x}{L} \Bigg] . \qquad \text{(ii)}$$

Integrating equation (ii) twice with respect to x and observing that at the ends $x = 0$, $x = L$, $\frac{\partial^2 w}{\partial x^2} = 0$, the following is obtained:

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{\varphi=0} = \frac{1}{E h a^4} \Bigg[- \frac{L^2}{\pi^2} \sum_{m=1}^{\infty} \Big\{ \Big[\gamma^6_{1m} + 15\gamma^2_{1m}\beta^2_{1m} (\beta^2_{1m} - \gamma^2_{1m})$$

$$-\beta^6_{1m} \Big] C_{1m} + 2\gamma_{1m}\beta_{1m} \left[10\gamma^2_{1m}\beta^2_{1m} - 3(\gamma^4_{1m} + \beta^4_{1m}) \right] C_{2m}$$

$$+ \left[\gamma^6_{2m} + 15\gamma^2_{2m}\beta^2_{2m}(\beta^2_{2m} - \gamma^2_{2m}) - \beta^6_{2m} \right] C_{3m}$$

$$+ 2\gamma_{2m}\beta_{2m} \left[10\gamma^2_{2m}\beta^2_{2m} - 3(\gamma^4_{2m} + \beta^4_{2m}) \right] C_{4m} \Big\} \frac{1}{m^2} \sin \frac{m\pi x}{L}$$

$$+\sum_{m=1}^{\infty}\left\{\left(2a^2-\frac{L^2}{\pi^2 m^2}\right)\left[(\gamma^4_{1m}-6\gamma^2_{1m}\beta^2_{1m}+\beta^4_{1m})C_{1m}-4\gamma_{1m}\beta_{1m}(\gamma^2_{1m}-\beta^2_{1m})C_{2m}\right.\right.$$

$$\left.+(\gamma^4_{2m}-6\gamma^2_{2m}\beta^2_{2m}+\beta^4_{2m})C_{3m}-4\gamma_{2m}\beta_{2m}(\gamma^2_{2m}-\beta^2_{2m})C_{4m}\right]\sin\frac{m\pi x}{L}$$

$$+\left[(2+\mu)a^2-\frac{a^4m^2\pi^2}{L^2}\right](\gamma^2_{1m}-\beta^2_{1m})C_{1m}-2\gamma_{1m}\beta_{1m}C_{2m}+(\gamma^2_{2m}-\beta^2_{2m})C_{3m}$$

$$\left.\left.-2\gamma_{2m}\beta_{2m}C_{4m}\right]\sin\frac{m\pi x}{L}\right\}\Bigg]. \qquad \text{(iii)}$$

For simplicity, this equation may be represented as follows:

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{\phi=0}=\frac{1}{Eh\,a^4}\sum_{m=1}^{\infty}(K_{1m}C_{1m}+K_{2m}C_{2m}+K_{3m}C_{3m}+K_{4m}C_{4m})\sin\frac{m\pi x}{L}, \qquad \text{(iv)}$$

where

$$K_{1m}=\frac{L^2}{\pi^2}(\gamma^6_{1m}+15\gamma^2_{1m}\beta^2_{1m}(\beta^2_{1m}-\gamma^2_{1m})-\beta^6_{1m})\frac{1}{m^2}$$

$$+\ 2a^2-\frac{L^2}{\pi^2 m^2}\quad(\gamma^4_{1m}-6\gamma^2_{1m}\beta^2_{1m}+\beta^4_{1m})$$

$$+\left[(2+\mu)a^2-a^4\frac{\pi^2 m^2}{L^2}\right]\quad(\gamma^2_{1m}-\beta^2_{1m}),$$

$$K_{2m}=-\frac{L^2}{\pi^2}2\gamma_{1m}\beta_{1m}\left[10\gamma^2_{1m}\beta^2_{1m}-3(\gamma^4_{1m}+\beta^4_{1m})\right]\frac{1}{m^2}$$

$$-\ (2a^2-\frac{L^2}{\pi^2 m^2}\quad 4\gamma_{1m}\beta_{1m}(\gamma^2_{1m}-\beta^2_{1m})$$

$$-\left[(2+\mu)a^2-\frac{a^4m^2\pi^2}{L^2}\right]2\gamma_{1m}\beta_{1m}.$$

K_3 is the same as K_1 except γ_{1m}, β_{1m} are replaced by γ_{2m} and β_{2m} respectively, and

K_{4m} is the same as K_{2m} except γ_{2m} and β_{2m} replace γ_{1m} and β_{1m} respectively.

But this expression for $\left.\frac{\partial^2 w}{\partial x^2}\right|_{\phi=0}$ only accounts for the curvature of the flange due to the edge moments and forces. The change in curvature due to the pressure loading on the shell should be superposed on the above expression. The exact expression for the deflection of a cylindrical shell of finite length with axial stiffeners and loaded with internal pressure (deflection in a plane containing an axial stiffener and the center line of the cylinder) would be an extremely difficult expression to derive. This thesis is concerned primarily with stresses and deflections at points at considerable distances from the ends of the cylinder, and it can be shown (for the sizes of cylinders considered) that the assumption that pressure loading causes no change in curvature of the axial stiffener (horizontal flange) is an assumption which results in small error. We can take the extreme case of an infinitely long cylinder with uniform internal pressure and with axial stiffeners; then one can see that there will be no bending in the axial stiffeners. For cylinders of finite length, one can estimate the magnitude of error (in assuming no bending in the axial stiffeners) by assuming the axial stiffeners to be bent so as to conform to the deflection shape of the cylinder, and then by calculating the resulting bending stress in the axial stiffeners. The magnitude of this stress would indicate whether the bending of the stiffeners is significant. Consider Fig. 5. Consider the cylinder in terms of a beam on an elastic foundation,

infinite in length, built in at the end, and loaded with a uniform pressure. The curvature at a point x is

$$\frac{d^2y}{dx^2} = -4\frac{q\eta^2}{K} B_{\eta x} + \frac{q}{2\eta^2} A_{\eta x},$$

where

$$B_{\eta x} = e^{-\eta x} \sin \eta x, \qquad A_{\eta x} = e^{-\eta x} (\cos \eta x + \sin \eta x),$$

$$K = \frac{Eh}{a^2}, \qquad \eta = \frac{1.285}{\sqrt{ah}}.$$

A plot of $A_{\eta x}$ and $B_{\eta x}$ versus ηx will show that both decrease to zero before ηx exceeds 3. For the size of cylinders considered in this thesis, η is equal to or greater than .9; hence the curvature (and the axial bending) diminishes to a negligible quantity at distances of approximately three inches from the ends. Therefore for cylinders of lengths of 25 inches or greater, the axial bending would be negligible over the greater length of the cylinder. Hence the bending of the axial stiffeners due to uniform internal pressure will be neglected, and it will be assumed that the only change in curvature will be that given by equation (iv), or

$$\frac{d^2 w}{d x^2} = \frac{\partial^2 w}{\partial x^2}\bigg|_{\phi=0}. \tag{v}$$

Evaluation of V_s (See page 8)

$$
\begin{aligned}
V_s = \frac{aL}{4D_s} \sum_{m=1}^{\infty} \int_0^{\pi/2} \Big[& C^2_{1m} \; e^{-2\gamma_{1m}\phi} + C^2_{3m}\, e^{-2\gamma_{2m}\phi} \cos^2\beta_{2m}\phi \\
& + C^2_{2m}\, e^{-2\gamma_{1m}\phi} \sin^2\beta_{1m}\phi + C^2_{4m}\, e^{-2\gamma_{2m}\phi} \sin\beta_{2m}\phi \\
& + C_{1m} C_{2m}\, e^{-2\gamma_{1m}\phi} \sin 2\beta_{1m}\phi + C_{3m} C_{4m} e^{-2\gamma_{2m}\phi} \sin 2\beta_{2m}\phi \\
& + e^{-(\gamma_{1m}+\gamma_{2m})\phi} \Big\{ C_{1m} C_{3m} \left[\cos(\beta_{1m}+\beta_{2m})\phi + \cos(\beta_{1m}-\beta_{2m})\phi \right] \\
& + C_{2m} C_{4m} \left[\cos(\beta_{1m}-\beta_{2m})\phi - \cos(\beta_{1m}+\beta_{2m})\phi \right] \\
& + C_{1m} C_{4m} \left[\sin(\beta_{1m}+\beta_{2m})\phi - \sin(\beta_{1m}-\beta_{2m})\phi \right] \\
& + C_{2m} C_{3m} \left[\sin(\beta_{1m}+\beta_{2m})\phi + \sin(\beta_{1m}-\beta_{2m})\phi \right] \Big\} \Big] \, d\phi \, .
\end{aligned}
$$

Let $I_1 = \int^{\pi/2} e^{-2\gamma_{1m}\phi} \cos^2\beta_{1m}\phi \, d\phi$

$$
= \frac{\beta_{1m}}{4(\gamma^2_{1m}+\beta^2_{1m})} \left[\frac{2\gamma^2_{1m}+\beta^2_{1m}}{\gamma_{1m}\,\beta_{1m}} + e^{-\pi\gamma_{1m}} \left(\sin\pi\beta_{1m} - \frac{2\gamma^2_{1m}\cos^2\beta_{1m}\frac{\pi}{2} + \beta^2_{1m}}{\gamma_{1m}\,\beta_{1m}} \right) \right] . \qquad (1)
$$

Let $I_{1.5} \equiv I_1$, except that γ_{1m} and β_{1m} are replaced by γ_{2m} and β_{2m}, respectively. (2)

$$\text{Let } I_2 = \int_0^{\pi/2} e^{-2\gamma_{1m}\phi} \sin^2 \beta_{1m}\phi \, d\phi = \frac{\beta_{1m}}{4(\gamma^2_{1m} + \beta^2_{1m})} \left\{ \frac{\beta_{1m}}{\gamma_{1m}} - e^{-\pi\gamma_{1m}} \left[\sin \pi\beta_{1m} + \frac{2\gamma^2_{1m} \sin^2 \beta_{1m} \pi/2 + \beta^2_{1m}}{\gamma_{1m}\beta_{1m}} \right] \right\} \tag{3}$$

$I_{2.5} \equiv I_2$, except that γ_{1m} and β_{1m} are replaced by γ_{2m} and β_{2m}. (4)

$$I_3 = \int^{\pi/2} e^{-2\gamma_{1m}\phi} \sin 2\beta_{1m}\phi \, d\phi = \frac{\beta_{1m}}{2(\gamma^2_{1m} + \beta^2_{1m})} \left[1 - e^{-\pi\gamma_{1m}} \left(\frac{\gamma_{1m}}{\beta_{1m}} \sin \pi\beta_{1m} + \cos \pi\beta_{1m} \right) \right] . \tag{5}$$

$I_{3.5} \equiv I_3$, except that γ_{1m} and β_{1m} are replaced by γ_{2m} and β_{2m}, respectively.

$$I_4 = \int_0^{\pi/2} e^{-(\gamma_{1m} + \gamma_{2m})\phi} \cos \phi(\beta_{1m} - \beta_{2m}) \, d\phi$$

$$= \frac{\beta_{1m} - \beta_{2m}}{(\gamma_{1m} + \gamma_{2m})^2 + (\beta_{1m} - \beta_{2m})^2} \left\{ \frac{\gamma_{1m} + \gamma_{2m}}{\beta_{1m} - \beta_{2m}} + e^{-(\gamma_{1m} + \gamma_{2m})\pi/2} \left[\sin \frac{\pi}{2} (\beta_{1m} - \beta_{2m}) - \frac{\gamma_{1m} + \gamma_{2m}}{\beta_{1m} - \beta_{2m}} \cos \frac{\pi}{2} (\beta_{1m} - \beta_{2m}) \right] \right\} . \tag{7}$$

$I_{4.5} \equiv I_4$, except that $-\beta_{2m}$ is replaced by $+\beta_{2m}$. (8)

$$I_5 = \int^{\pi/2} e^{-(\gamma_{1m} + \gamma_{2m})\phi} \sin \phi(\beta_{1m} - \beta_{2m}) d\phi$$

$$= \frac{(\beta_{1m} - \beta_{2m})}{(\gamma_{1m} + \gamma_{\gamma m})^2 + (\beta_{1m} - \beta_{2m})^2} \left\{ 1 - e^{-(\gamma_{1m} + \gamma_{2m})\frac{\pi}{2}} \left[\cos \frac{\pi}{2}(\beta_{1m} - \beta_{2m}) \right. \right.$$

$$\left. \left. + \frac{\gamma_{1m} + \gamma_{2m}}{\beta_{1m} - \beta_{2m}} \sin \frac{\pi}{2} (\beta_{1m} - \beta_{2m}) \right] \right\} \quad (9)$$

$I_{5.5} \equiv I_5$, except that $-\beta_{2m}$ is replaced by $+\beta_{2m}$. (10)

Then

$$V_s = \frac{aL}{4D_s} \sum_{m=1}^{\infty} \left[I_1 C^2_{1m} + I_{1.5} C^2_{3m} + I_2 C^2_{2m} + I_{2.5} C^2_{4m} \right.$$

$$+ I_3 C_{1m} C_{2m} + I_{3.5} C_{3m} C_{4m} + (I_4 + I_{4.5}) C_{1m} C_{3m}$$

$$+ (I_4 - I_{4.5}) C_{2m} C_{4m} + (I_{5.5} - I_5) C_{1m} C_{4m}$$

$$\left. + (I_5 + I_{5.5}) C_{2m} C_{3m} \right].$$

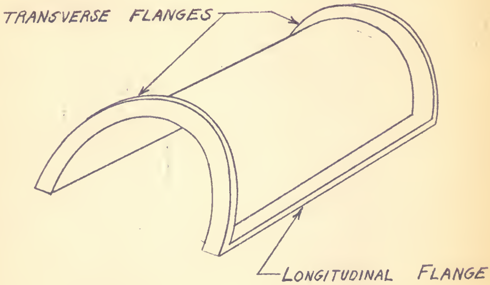


Fig. 1 Cylindrical shell with longitudinal and transverse flanges.

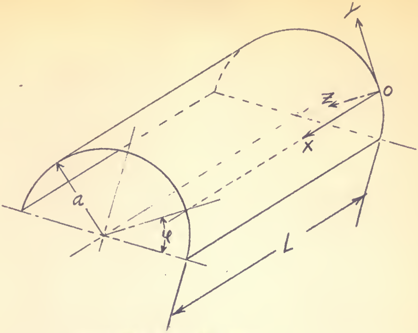


Fig. 2 Coordinate system used.

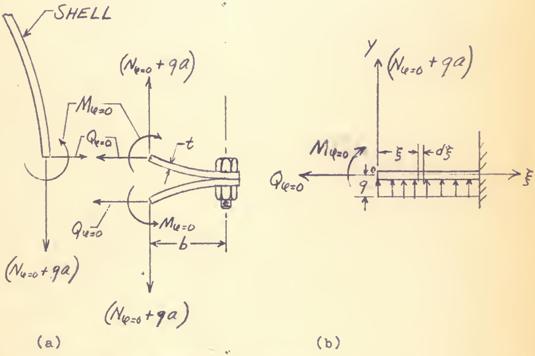


Fig. 3 (a) Loading of flanges.
(b) Analytical model of flange.

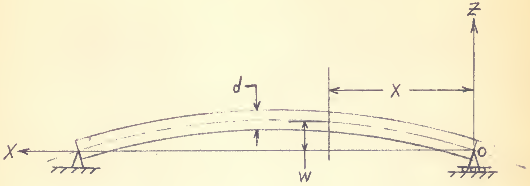


Fig. 4 View of longitudinal flange in a plane containing the center line of the cylindrical shell. The ends will be considered simply supported.

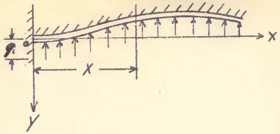


Fig. 5 Semi-infinite beam on elastic foundation.

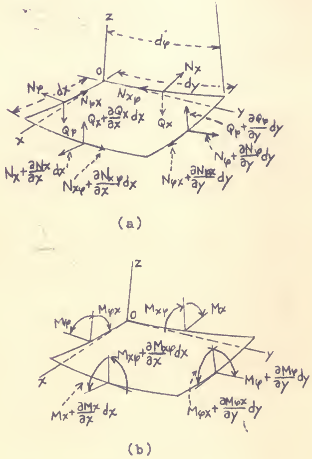
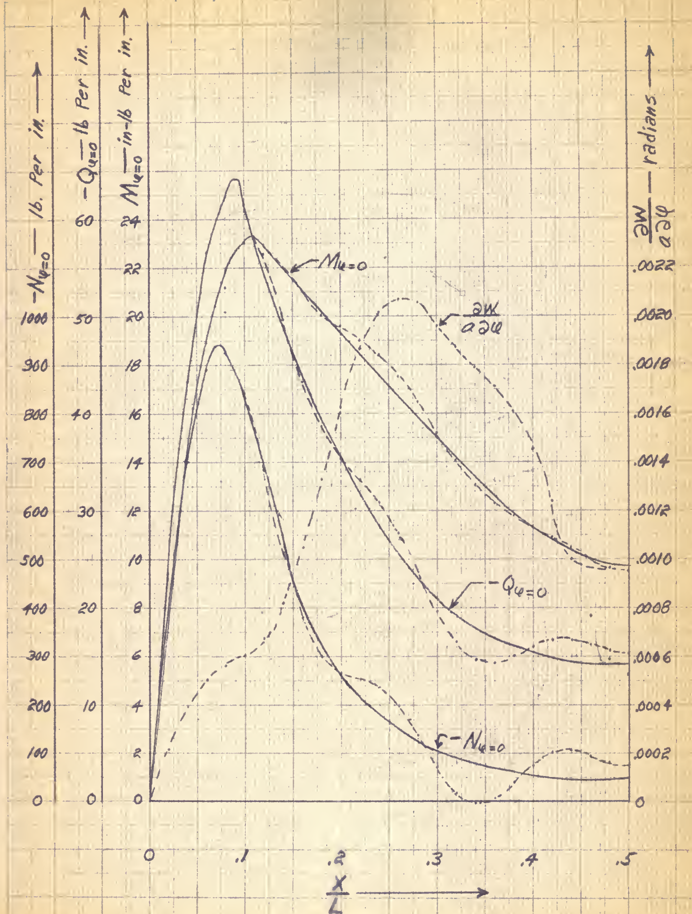


Fig. 6 (a) Forces
(b) Moments



$-N_{\varphi=0}$ — lb. Per in. →
$-Q_{\varphi=0}$ — lb Per in. →
$M_{\varphi=0}$ — in-lb Per in. →
$\frac{\partial w}{a\partial\varphi}$ — radians →
$M_{\varphi=0}$
$\frac{\partial w}{a\partial\varphi}$
$-Q_{\varphi=0}$
$-N_{\varphi=0}$
1000
900
800
700
600
500
400
300
200
100
0
60
50
40
30
20
10
24
22
20
18
16
14
12
10
8
6
4
2
.0022
.0020
.0018
.0016
.0014
.0012
.0010
.0008
.0006
.0004
.0002
.1
.2
.3
.4
.5
$\frac{X}{L}$ →

Fig. 7

Table 1 — "I" VALUES

m	I_1	$I_{1.5}$	I_2	$I_{2.5}$	I_3	$I_{3.5}$	I_4	$I_{4.5}$	I_5	$I_{5.5}$
1	.14515	.21417	.0113020	.1592086	.0550472	.1362548	.189170	.11111	-.078473	.110648
3	.08030	.12176	.0050231	.0849387	.0275000	.0791592	.106334	.067663	-.039225	.059963
5	.06011	.092618	.0031336	.0607554	.0188970	.0622211	.080506	.054343	-.026[illegible]	.043746
7	.04917	.077246	.0021414	.0473639	.0141910	.0532042	.066482	.047280	-.020311	.034659
9	.04199	.067535	.0015261	.0384042	.0111140	.0473020	.057265	.042680	-.015928	.028488

Table 2 — "K" VALUES

m	K_{1m}	K_{2m}	K_{3m}	K_{4m}	
1	123,995	113,706	-124,013	134,639	
3	372,917	336,358	-373,211	406,802	
5	622,654	532,228	-625,647	701,765	
7	872,452	700,199	-885,097	1,018,217	
9	1,121,122	839,492	-1,156,685	1,354,781	

TABLE 3 – COEFFICIENTS OF C_{1m}, C_{2m}, C_{3m}, AND C_{4m} IN EQUATIONS 33a, 34, 35, AND 36, Where

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix} \begin{vmatrix} C_{1m} \\ C_{2m} \\ C_{3m} \\ C_{4m} \end{vmatrix} = \begin{vmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{vmatrix}.$$

m ↓	α_{11}	α_{21}	α_{31}	α_{41}	α_{12}	α_{22}	α_{32}	α_{42}	α_{13}	α_{23}	α_{33}	α_{43}	α_{14}	α_{24}	α_{34}	α_{44}
1	1.0761 ~~4.3780~~	.3243 ~~1.1486~~	.7844 ~~3.6456~~	.7701 ~~3.0396~~	.3243 ~~1.1486~~	.2099 ~~.6749~~	$\overline{.04415}$ ~~.04175~~	.4082 ~~1.1341~~	.7844 ~~3.6456~~	$\overline{.04415}$ ~~.04175~~	1.5230 ~~6.2655~~	.2528 ~~1.3783~~	.7701 ~~3.0396~~	.4082 ~~1.4941~~	.2528 ~~1.3789~~	1.1880 ~~4.8321~~
3	~~6.1279~~ 1.9876	~~3.9263~~ 1.4166	~~$\overline{1.5958}$~~ $\overline{.9454}$	~~5.6434~~ 1.9201	~~3.9263~~ 1.4166	~~3.3609~~ 1.2512	~~$\overline{3.2561}$~~ $\overline{1.2481}$	~~1.4211~~ 1.5929	~~$\overline{1.5958}$~~ $\overline{.9454}$	~~$\overline{3.2561}$~~ $\overline{1.2481}$	~~7.2969~~ 2.2874	~~$\overline{3.2101}$~~ $\overline{1.3459}$	~~5.6434~~ 1.9201	~~1.4211~~ 1.5929	~~$\overline{3.2101}$~~ 1.3459	~~7.0128~~ 2.3075
5	4.5369	3.5553	$\overline{3.7672}$	4.7449	3.5553	3.0772	$\overline{3.4460}$	4.1006	$\overline{3.7672}$	$\overline{3.4460}$	4.8106	$\overline{4.4340}$	4.7449	4.1006	$\overline{4.4340}$	5.6654
7	8.5193	6.4639	$\overline{7.9531}$	9.5467	6.4639	5.3129	$\overline{6.4827}$	7.7315	$\overline{7.9531}$	$\overline{6.4827}$	2.9026	$\overline{9.3600}$	9.5467	7.7315	$\overline{9.3600}$	11.4166
9	13.8950	9.8888	$\overline{13.6344}$	16.1472	9.8888	7.6398	$\overline{10.2081}$	12.2836	$\overline{13.6344}$	$\overline{10.7081}$	14.7531	$\overline{16.4492}$	16.1472	12.2836	$\overline{16.4492}$	19.9140

m ↓	$+A_1$	A_2	A_3	A_4
1	$\bar{3}.4759$	$\bar{2}.4134$	$\bar{8}.3364$	$\bar{2}.4145$
3	.87998	$\bar{2}.4044$	$\bar{3}.9974$	$\bar{2}.4070$
5	2.0158	$\bar{2}.3639$	$\bar{2}.9028$	$\bar{2}.3866$
7	2.7004	$\bar{2}.3540$	$\bar{2}.2801$	$\bar{2}.3545$
9	3.7417	$\bar{2}.3160$	$\bar{7}.8220$	$\bar{2}.3108$

AN APPROXIMATE SOLUTION FOR THE BENDING
OF A CYLINDRICAL SHELL WITH TWO
LONGITUDINAL FLANGES AND LOADED
WITH INTERNAL PRESSURE

by

JOHN ATKINS MAYHALL

B. S., University of Alabama, 1951

AN ABSTRACT OF
A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE COLLEGE
OF AGRICULTURE AND APPLIED SCIENCE

1958

The object of this thesis was the mathematical determination of forces, moments, and deformations in a cylindrical shell comprised of two cylindrical halves bolted together along longitudinal flanges, the two halves having transverse flanges at the ends which can afford assembly to adjoining shells. The shell is subjected to uniform internal pressure.

Simplifications were made to the general equations which describe the bending of a laterally-loaded cylindrical shell, and the resulting simplified equations were used in conjunction with the principle of least work. The strain energy due to bending of the shell and longitudinal flanges was summed and then minimized, yielding expressions for the forces, moments and deflections in the shell and longitudinal flanges. A numerical example was included to demonstrate the use of the developed method.